Experiments in Thermal Conductivity and Mean Free Path

Determining the Diameter of an “Air” Molecule

Equipment Required

- Vacuum system capable of reaching approximately 100 milliTorr or below. The system must have a pump isolation valve and fine leak.
- Vacuum gauge cable of covering 0.1 to 20 Torr (MKS microPirani is referenced).
- Hot wire sensor device and power supply.
- Vernier hardware and software (or equivalent) for measuring and recording pressure (e.g. microPirani gauge) and hot wire sensor voltage.

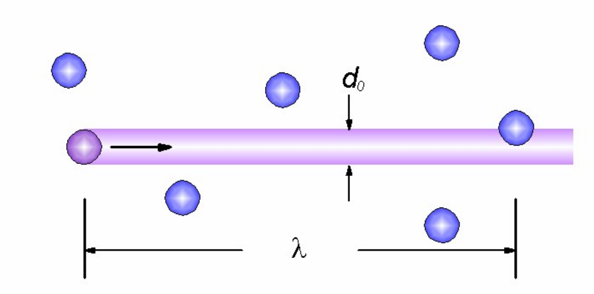
Objectives

In this exercise we will:

- Examine the relationship between pressure and thermal conductivity
- Exploit the relationship between mean free path and thermal conductivity to calculate the size of air (i.e. a mix of nitrogen/oxygen/water vapor) molecules
- Demonstrate how one major class of indirect vacuum gauge functions

Theory Behind the Exercise

The mean free path of a molecule is defined as the average distance between collisions of the molecules. At higher pressures the mean free path is shorter because of the higher density. As pressure is reduced the mean free path increases.



A simplified representation is shown in the figure above. If a molecule of diameter d_0 is moving at its thermal velocity through a gas, the distance that the molecule can travel before a collision occurs is related to the diameter of the molecules in the mixture and the number density, *n*, of the gas molecules. The number density is derived from the relationship where Avogadro’s number of molecules occupy 22.4 liters at 760 Torr and 273K (0 °C).

If the incident molecule in the figure is the only one in motion, the mean free path would be:

$$\lambda = \frac{1}{\pi d_0^2 n}$$

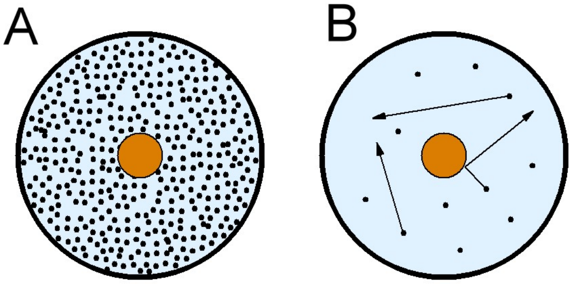
However, all of the molecules are in motion so the equation becomes a little more complicated:

$$\lambda = \frac{1}{\sqrt{2}\pi d_0^2 n}$$

In this exercise we will calculate d 0 through an indirect method of determining the mean free path.

Basis of the Experimental Method

Consider a configuration in which a heated filament is housed within a metallic shell that is held at ambient temperature and where the diameter of the filament is much smaller than the distance to the shell. At normal pressures we have the situation shown in “A” of the figure below.



Here the mean free path of the gas molecules is much smaller than the diameter of the filament. As a result, a majority of the gas molecules are impinging on the shell and assume an energy corresponding to the ambient temperature. Molecules that are heated by the filament will depart the filament with a correspondingly higher energy but will collide with the cooler molecules and lose energy. The vast majority of molecules leaving the filament will also return to the filament after a collision with a cooler molecule because of the short mean free path. The net effect of this is that the filament is transferring much of its thermal energy to ambient temperature molecules and this has a cooling effect on the filament.

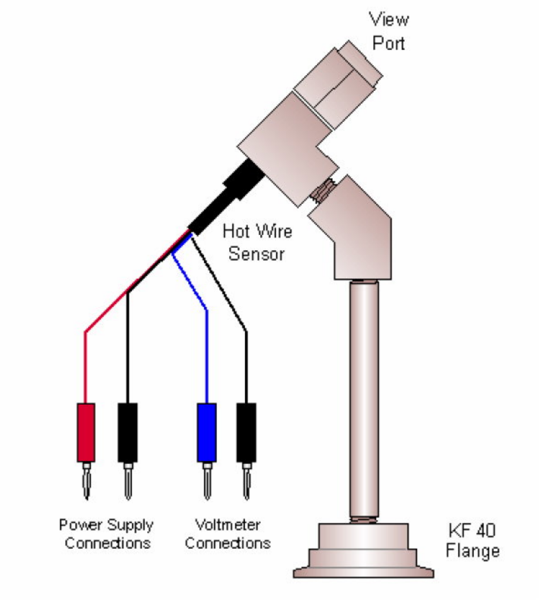
As the pressure is decreased the mean free path will eventually become as long as the filament diameter. At this point fewer molecules will collide with the filament thus decreasing the heat flux from the filament. As a result, the filament’s temperature will increase. This situation is depicted in “B” of the figure above.

There is a non-linear transition between the first condition (mean free path << filament diameter) and the second where the mean free path is equal to or greater than the filament diameter. At this point the relationship between thermal conductivity and pressure becomes linear.

At very low pressures (< 1 milliTorr) so few molecules collide with the filament that the only heat conduction is via the filament supports and through radiation. Neither of these is a function of pressure.

Set Up Procedure

In this exercise we will use a small device (a 2 volt model engine glow plug) with a filament whose resistance varies with temperature. The hotter the filament, the higher the resistance. The apparatus is designed to supply a constant current (nominally 1 amp) to the filament. By slowly varying the pressure in the chamber we can record the voltage across that filament against pressure. The device is shown in the figure below.



The sensor consists of a thin (for example 0.0045 inch or 114 micron diameter) resistance wire that is mounted within a metal shell. There are 2 pairs of wire: one for the filament power, the other for the voltage sense.

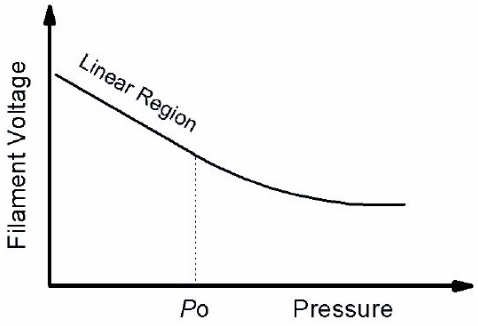
1. For the example shown, Install the device directly on the vacuum system's KF40 port. Orient it so that it is easy to look into the glass view port at the top of the instrument.
2. Attach the red and one of the black leads to the corresponding jacks on the power supply.
3. Attach the green and the other black lead to the Vernier voltage probe clips.
4. To improvise a leak, attach about 6 inches of 1/4-inch inside diameter silicone (milky white color) tubing to an auxiliary port on the vacuum system. Using a pinch clamp, close the end of tube. Since silicone is very permeable to gases, this tube will serve as a slow leak.
5. Set up the Vernier Logger Pro software as shown below with the voltage across the filament on the y axis and pressure (as measured by the microPirani gauge) on the x axis.
6. If desired, the approximate filament temperature may be plotted. This is possible since the filament behaves as a resistance temperature device (RTD). Plotting this requires that the temperature characteristic as a function of filament voltage be entered into Logger Pro as an equation. The relationship is:

   T (°C) = 2592 x V -794

Taking the Data

1. With the system at atmospheric pressure, plug in the power supply.
2. Close the VPAL-A's exhaust valve and vent valve.
3. Ensure that the microPirani gauge is reading atmospheric pressure. Span the gauge if necessary.
4. Put on safety glasses
5. Turn on the vacuum pump.
6. While looking into the view port of the sensor, begin to reduce the pressure by cracking the foreline valve. At some point in the pump down the filament will begin to glow. Make a note of the indicated pressure.
7. Continue to pump to base pressure (0.10 Torr or below).
8. When at base pressure start data collection with the Vernier software. A point should appear denoting filament voltage and pressure.
9. Close the foreline valve and allow the pressure to rise. Let the pressure rise until the curve becomes flat.
10. Stop and save the data collection, vent the manifold, turn off the pump and open the exhaust valve to bring the entire system up to atmosphere. Turn off the power supply.

The curve should be of the form shown in the figure below.



During pump down, at what pressure did the filament begins to glow?

At what pressure did the filament stop glowing?

Is there a relationship between the glowing of the filament and the curve that you have just plotted?

Analysis

Carefully examine the curve. (It may be necessary to apply an averaging function in the software to smooth out variations in the curve.) Going from low pressure to high, determine at what pressure the curve departs from being linear. This is P_0 .

Calculate n in units of m^{-3} at P_0 .

State the mean free path λ at P_0 in meters and explain where this number came from

Calculate the diameter of the gas molecule using the expression

$$\lambda = \frac{1}{\sqrt{2}\,\pi d_0^2 n}$$

Further Discussion

The hot wire sensor operates in much the same manner as the microPirani gauge that is on the vacuum system. In the process of doing the exercise we have calibrated the sensor and it may be used as a simple Pirani gauge.

The commercial gauge is able to measure to higher pressures than the simple hot wire sensor. Do some research to determine why this is the case.